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Automorphisms of $K3$ surfaces and Enriques involutions.

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Conjugacy Classes of Free Involutions

Let X be a $K3$ surface. It happens that X covers two or more non-isomorphic Enriques surfaces. The first example has been found by S. Kondo. Then the problem is whether one can determine all the Enriques quotients of a given $K3$ surface X, or equivalently, all the conjugacy classes of free involutions in $\mathrm{Aut}(X)$. I start with the following finiteness result.

Theorem: In $\mathrm{Aut}(X)$ there are only finitely many conjugacy classes of finite subgroups. In particular, there are only finitely many Enriques quotients of X modulo isomorphisms.

For the latter statement, I have two proofs. One, suggested by an anonymous referee uses Borel's reduction theory. The other is lattice theoretic, and can be sharpened so that we can count the exact number of non-isomorphic Enriques quotients of X.

Theorem: When $\rho(X) = 11$, we can classify the Neron-Severi group $NS(X)$ of X and compute the exact number of non-isomorphic Enriques quotients. For example, $U(2) \oplus E_8(2) \oplus \langle -8p_1^{e_1} \cdots p_l^{e_l} \rangle$ (p_i are distinct odd primes) can appear as $NS(X)$ and in this case the number is equal to 2^{l+10}.

Corollary: There doesn't exist the universal upper bound for the number of non-isomorphic Enriques quotients.

A Kummer surface of product type is a $K3$ surface which is the minimal desingularization of $E \times F/\{\pm 1\}$, where E and F are elliptic curves. They move in a two-dimensional family. There has been found two types of Enriques involutions on a Kummer surface of product type. I proved that they are the all.

Theorem: If X is a very general Kummer surface of product type, then X has exactly 15 non-isomorphic Enriques quotients. They are constructed by Lieberman involutions and Kondo-Mukai involutions.

A jacobian Kummer surface is a $K3$ surface which is the minimal desingularization of $J(C)/\{\pm 1\}$, where $J(C)$ is the jacobian variety of a smooth curve C of genus 2. I'm now trying to determine the Enriques involutions on X. The number is already computed, and the geometric constructions are conjectured as follows.

Conjecture: If X is a very general jacobian Kummer surface, then X has exactly 31 non-isomorphic Enriques quotients. They are constructed by known involutions, namely 10 of which are switches, 6 are involutions associated with Weber hexads, 15 are involutions associated with Göpel tetrads.

Remark: S. Mukai has encountered the involutions associated with Göpel tetrads in his research of automorphisms of Enriques surfaces. The conjecture above is due to him.

Finite subgroups of automorphism groups of generic Enriques surfaces.

The automorphism group of a generic Enriques surface Y was computed by W. Barth and C. Peters. It is the 2-congruence subgroup of the orthogonal group of the Enriques lattice. They also counted the number of non-isomorphic Horikawa representations. Each Horikawa representation induces an involution on Y. (They are of rank 8 in Mukai's term.) Using the structure of orghogonal groups, I can show the following.

Theorem: Let Y be a generic Enriques surface. Then any nontrivial finite subgroup of $\mathrm{Aut}(Y)$ is of order 2.

Automorphisms of $K3$ surfaces in characteristic 11.

Recently I. Dolgachev and J. H. Keum extended Mukai's classification of finite symplectic automorphism groups of $K3$ surfaces over $\mathbb{C}$ to positive characteristics ≥ 13. They also classified the possible groups in characteristic 11. Their question is: are there any symplectic automorphism with an isolated fixed point?

Proposition: There exists such.

Such an automorphism is peculiar to positive characteristic. The proof shows only its existence. It uses Roger's upper bound of center density of sphere packings.